Solution Extraction from Long-distance Resolution Proofs

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KBS Group, Institute of Information Systems, Vienna University of Technology

International Workshop on Quantified Boolean Formulas

FAKULTÄT FÜR !NFORMATIK

Faculty of Informatics

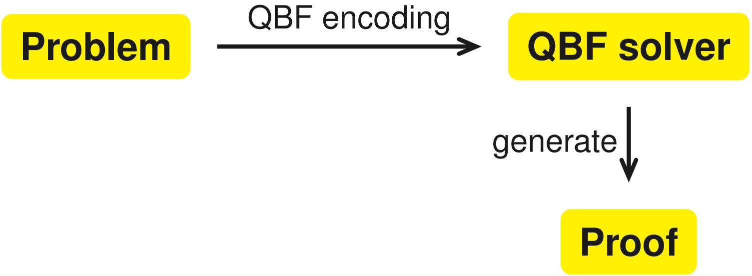
Overview

Problem

Overview

Problem $\xrightarrow{\text{QBF encoding}}$ **QBF solver**

Overview



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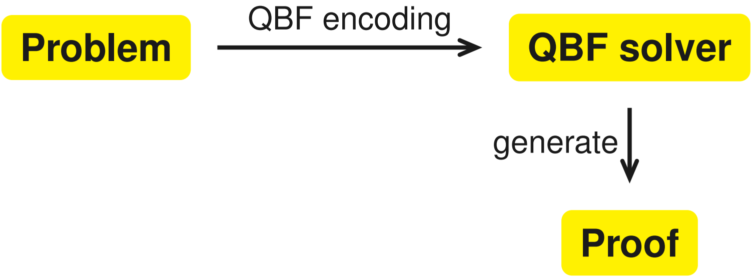

E.g. Q-Resolution by DepQBF [Lonsing and Biere, 2010]

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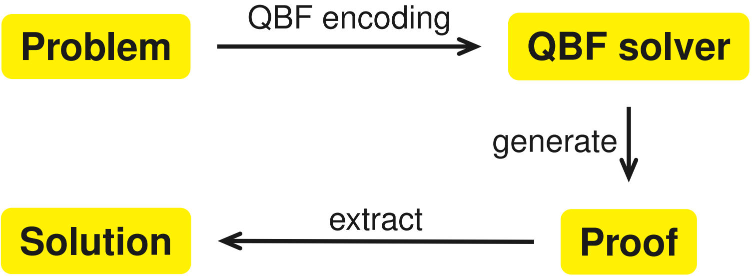

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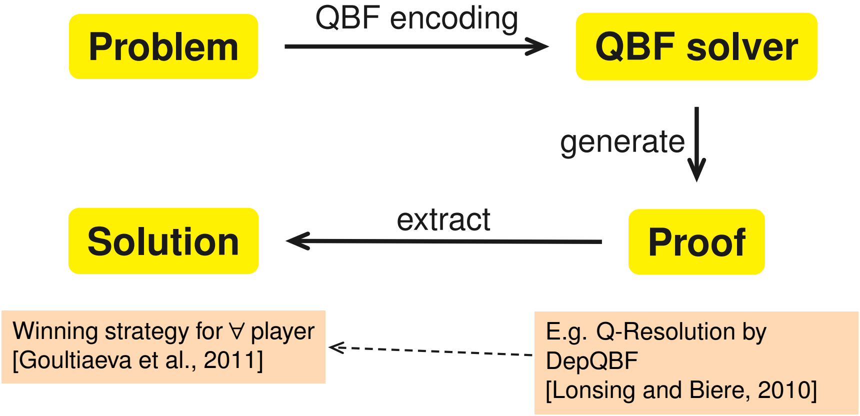
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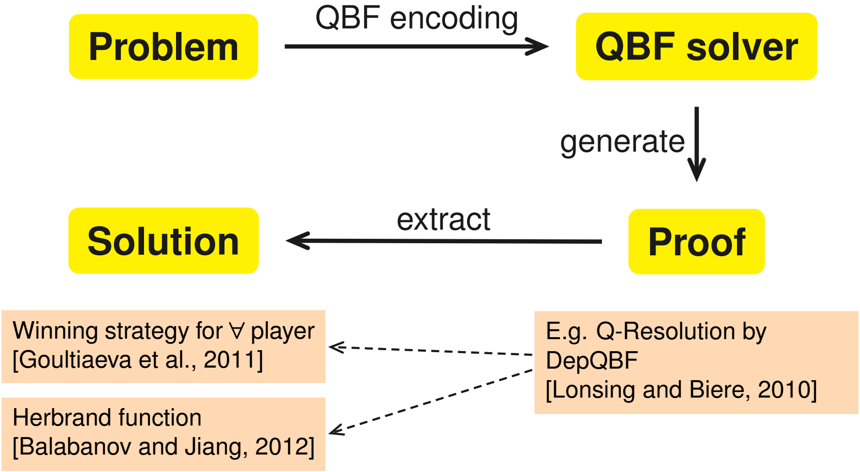

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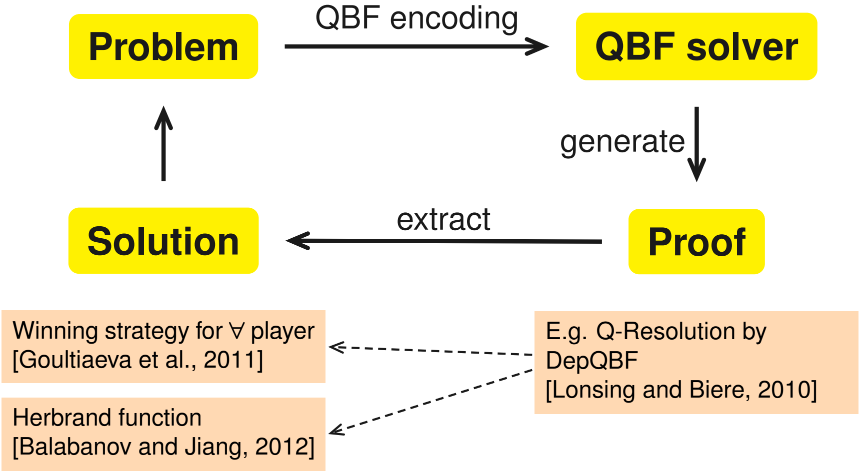

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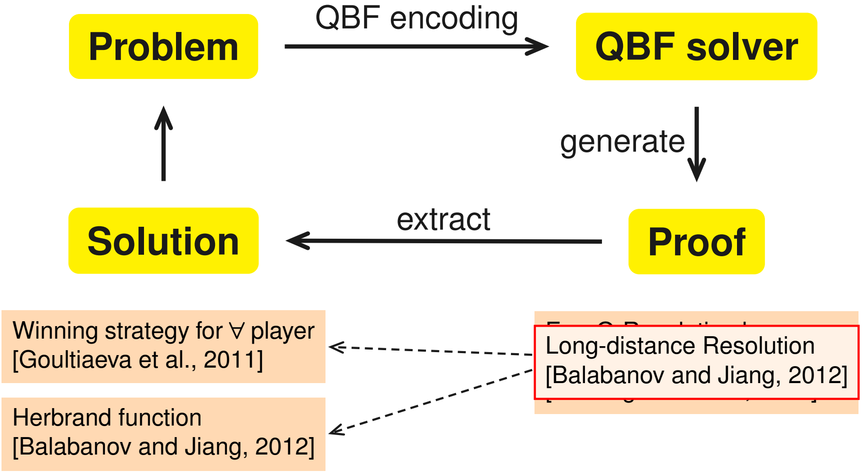

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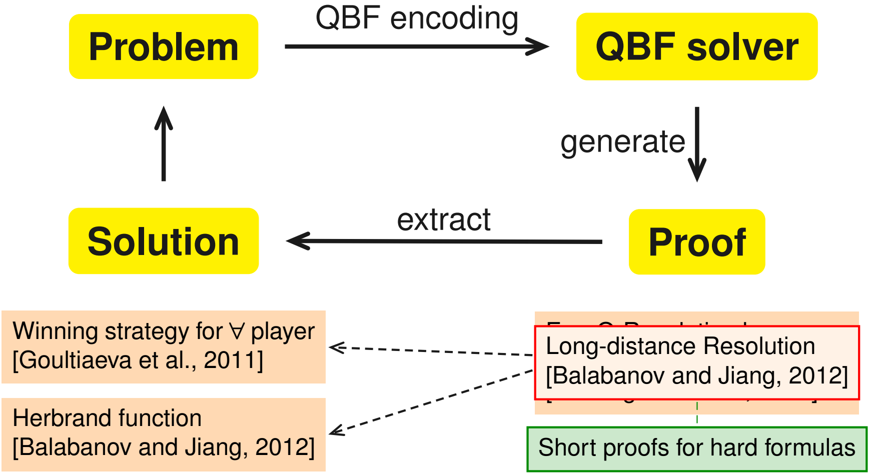
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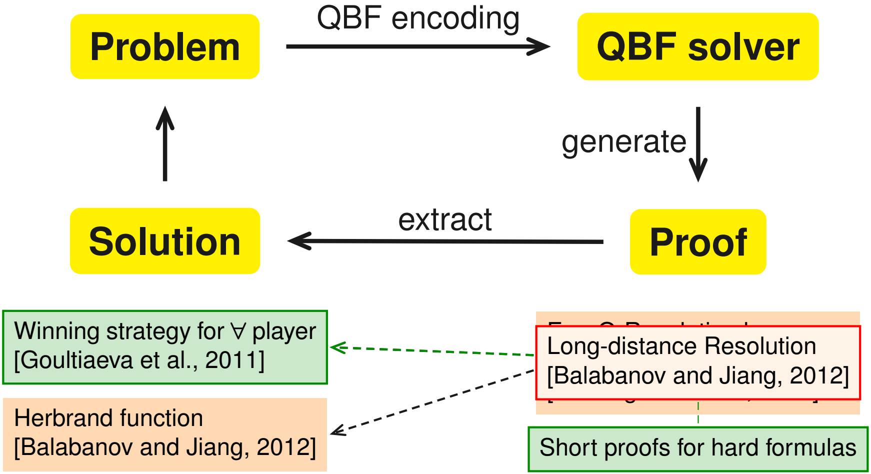

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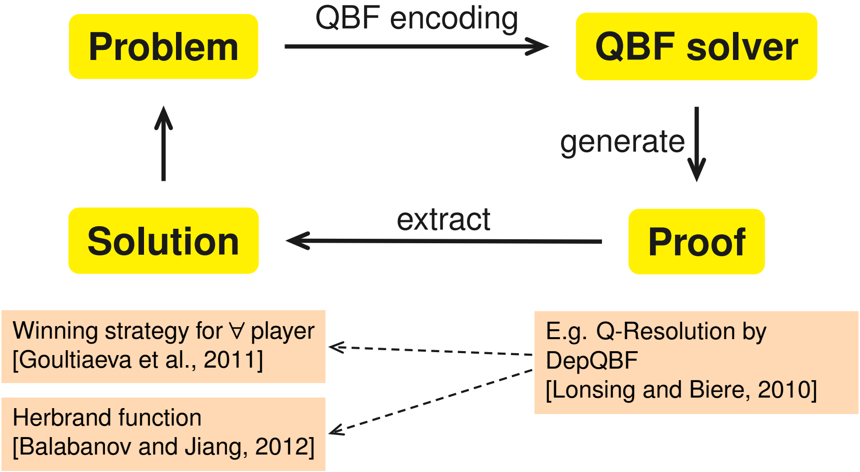
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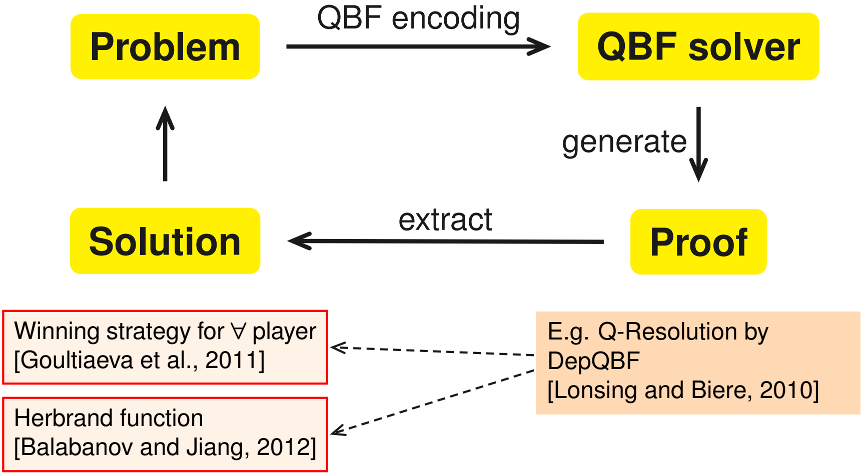
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Solution Representations

Winning strategy [Goultiaeva et al., 2011]

- Game-theoretic view
- Controller computes move for the $\forall$ player according to previous moves of the $\exists$ player
- Runtime polynomial *in the size of the refutation*

Solution Representations

Winning strategy [Goultiaeva et al., 2011]

- Game-theoretic view
- Controller computes move for the $\forall$ player according to previous moves of the $\exists$ player
- Runtime polynomial *in the size of the refutation*

Herbrand functions [Balabanov and Jiang, 2012]

- Function for each $\forall$ variable depending on $\exists$ variables to its left
- Replacing each $\forall$ variable by its function renders the matrix unsatisfiable
- Runtime polynomial *in the size of the refutation*

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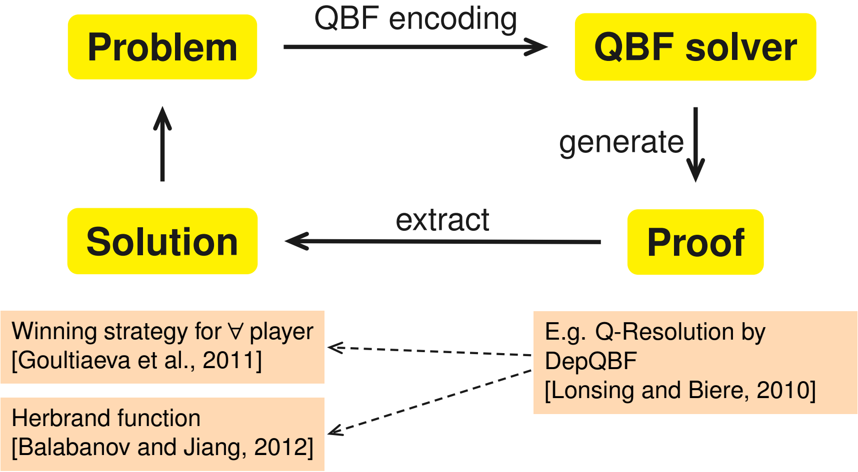
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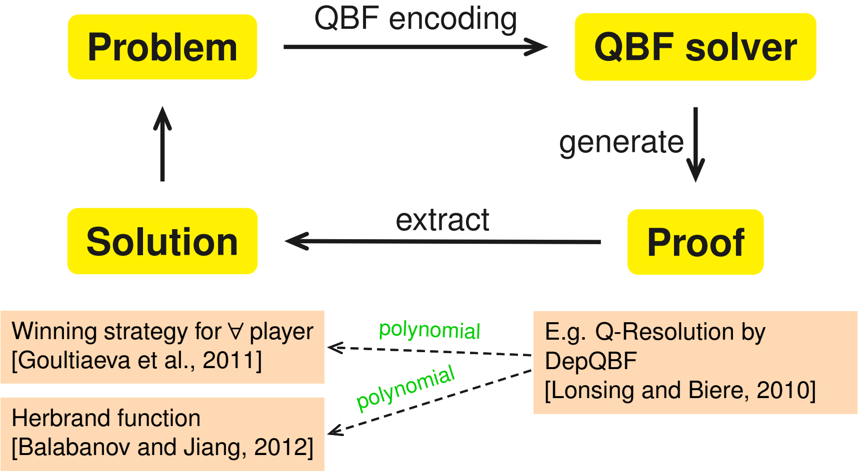
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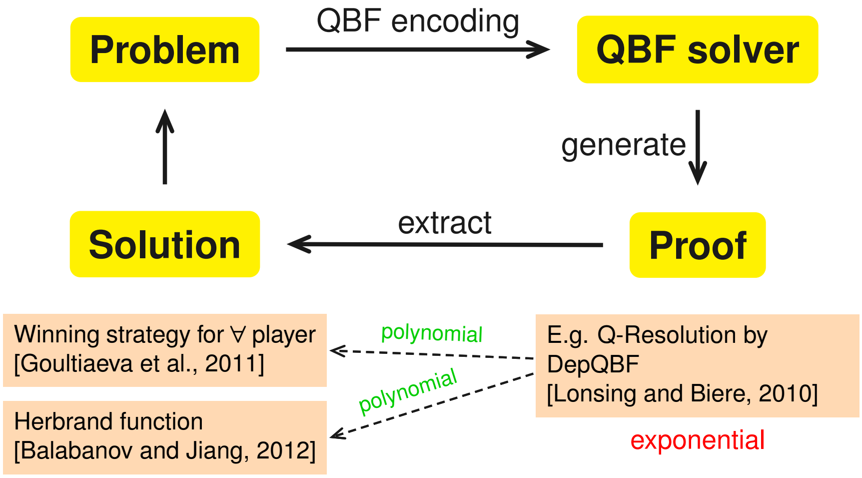
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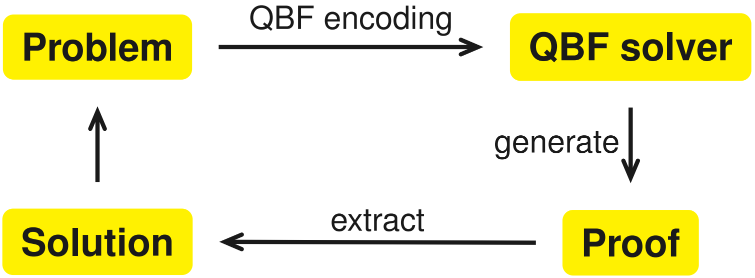

Winning strategy for ∀ player
[Goultiaeva et al., 2011]

Herbrand function
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Long-distance Resolution
[Balabanov and Jiang, 2012]

Long-distance Resolution (LDQ-Resolution)

Q-resolution with the following additional derivation rules

$$\frac{K^l \vee p \vee x \qquad K^r \vee \bar{p} \vee \bar{x}}{K^l \vee K^r \vee x^*} \quad \mathbf{lev}(p) < \mathbf{lev}(x)$$

$$\frac{K^l \vee p \vee x \qquad K^r \vee \bar{p} \vee x^*}{K^l \vee K^r \vee x^*} \quad \mathbf{lev}(p) < \mathbf{lev}(x)$$

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[Balabanov and Jiang, 2012]

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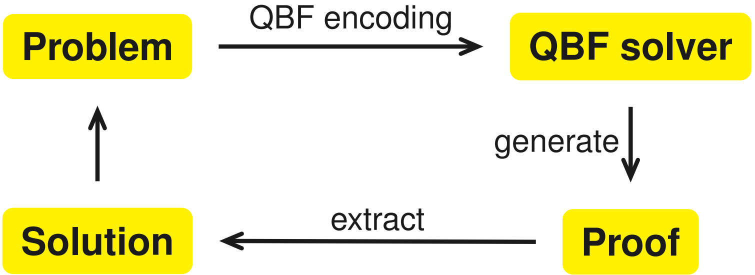

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Short proofs for hard formulas

Short Proofs for Hard Formulas

Family $(\varphi_t)_{t \geq 1}$ of QBFs in PCNF with prefix

$$\exists d_0 d_1 e_1 \forall x_1 \exists d_2 e_2 \forall x_2 \exists d_3 e_3 \cdots \forall x_{t-1} \exists d_t e_t \forall x_t \exists f_1 \cdots f_t$$

and matrix:

$$\begin{array}{llllll}
K_0 & \overline{d_0} & K_1 & d_0 \vee \overline{d_1} \vee \overline{e_1} & \\
K_{2j} & d_j \vee \overline{x_j} \vee \overline{d_{j+1}} \vee \overline{e_{j+1}} & K_{2j+1} & e_j \vee x_j \vee \overline{d_{j+1}} \vee \overline{e_{j+1}} & j = 1, \ldots, t-1 \\
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- Any Q-refutation for φ_t is exponential. [Kleine Büning et al., 1995]

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- Q-resolution plus resolution over $\forall$ variables yields polynomial refutations. [Van Gelder, 2012]

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- Any Q-refutation for φ_t is exponential. [Kleine Büning et al., 1995]
- Q-resolution plus resolution over ∀ variables yields polynomial refutations. [Van Gelder, 2012]
- Polynomial LDQ-refutation is possible. [This work]

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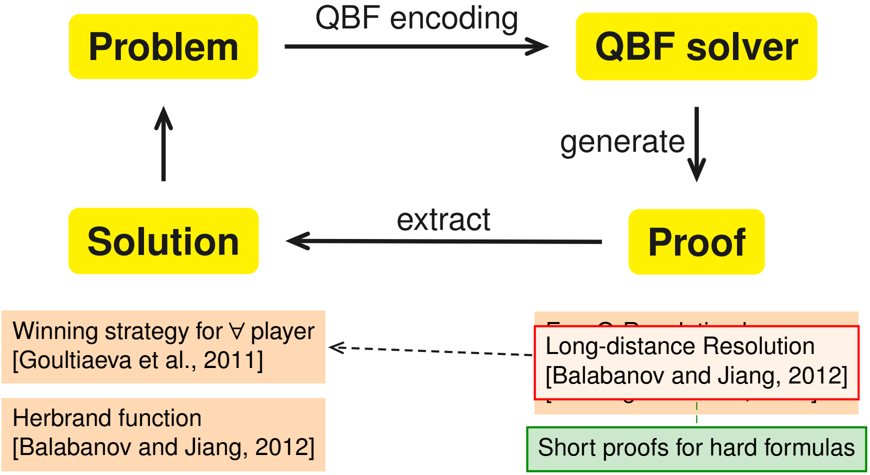
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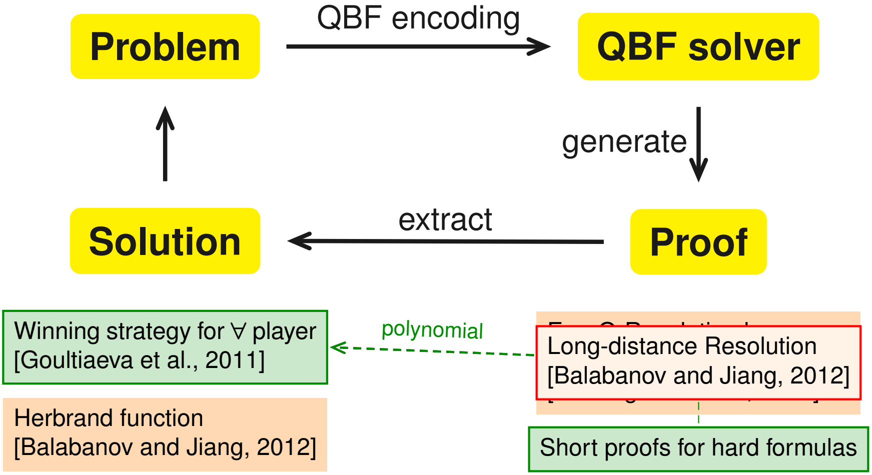
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Strategy Extraction from LDQ-Refutations

$\exists a \; \forall x \; \exists b \; \forall y \; \exists c$

$(a, x, b, y, c) \qquad (\bar{a}, \bar{y}) \qquad (\bar{c}) \qquad (\bar{b})$

Strategy Extraction from LDQ-Refutations

$\exists a\ \forall x\ \exists b\ \forall y\ \exists c$

LDQ-Resolution

$(a, x, b, y, c) \qquad (\bar{a}, \bar{y}) \qquad (\bar{c}) \qquad (\bar{b})$

$$(x, b, y^*, c)$$

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$\exists \boldsymbol{a}\ \forall \boldsymbol{x}\ \exists \boldsymbol{b}\ \forall \boldsymbol{y}\ \exists \boldsymbol{c}$

LDQ-Resolution

$$\begin{tikzcd}[column sep=small]
(a, x, b, y, c) \arrow[dr] & & (\bar{a}, \bar{y}) \arrow[dl] & & (\bar{c}) \arrow[ddll] & & (\bar{b}) \\
& (x, b, y^*, c) \arrow[dr] & & & & & \\
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& & (x, b) & & &
\end{tikzcd}$$

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$$
\begin{tikzcd}
(a, x, b, y, c) \arrow[dr] & & (\bar{a}, \bar{y}) \arrow[dl] & (\bar{c}) \arrow[ddl] & (\overline{\textcolor{red}{b}}) \arrow[ddddl] \\
& (x, b, y^*, c) \arrow[dr] & & & \\
& & (x, b, y^*) \arrow[d] & & \\
& & (x, \textcolor{red}{b}) \arrow[dr] & & \\
& & & (x) &
\end{tikzcd}
$$

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& & (x, b, y^*) \arrow[d] & & & \\
& & (x, b) \arrow[dr] & & & \\
& & & ({\color{blue} x}) \arrow[d] & & \\
& & & \bot & &
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$$

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& & & \bot & & &
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$\color{orange}{\exists a}$ $\forall x$ $\exists b$ $\forall y$ $\exists c$

Strategy Extraction

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& & & \bot &
\end{tikzcd}
$$

Strategy Extraction from LDQ-Refutations

$\exists a \; \forall x \; \exists b \; \forall y \; \exists c$

Strategy Extraction

$$(a, x, b, y, c) \qquad (\bar{a}, \bar{y}) \qquad (\bar{c}) \qquad (\bar{b})$$

$$(x, b, y^*, c)$$

$$(x, b, y^*)$$

$$(x, b)$$

$\exists$ assignment $= \{\bar{a}\}$

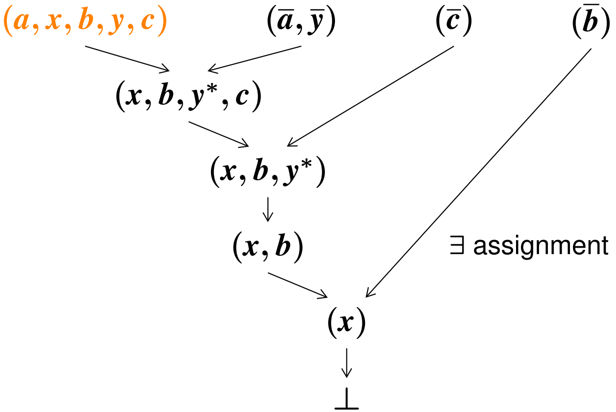
$$(x)$$

$$\bot$$

Strategy Extraction from LDQ-Refutations

$\exists a\ \forall x\ \exists b\ \forall y\ \exists c$

Strategy Extraction



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 & (x, b, y^*) \arrow[d] & & \\
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\begin{tikzcd}
(x, b, y, c) \arrow[dr] & & (\top) \arrow[dl] & & (\bar{c}) \arrow[ddll] & & (\bar{b}) \arrow[ddddlll] \\
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Strategy Extraction from LDQ-Refutations

Strategy Extraction

$\exists a \; \forall x \; \exists b \; \forall y \; \exists c$

$$\begin{tikzcd}
(x, b, y, c) \arrow[dr] & & (\top) \arrow[dl] & (\bar{c}) \arrow[ddl] & (\bar{b}) \arrow[ddddl] \\
& \color{red}{(x, b, y, c)} \arrow[dr] & & & \\
& & (x, b, y^*) \arrow[d] & & \\
& & (x, b) \arrow[dr] & & \\
& & & (x) \arrow[d] & \\
& & & \bot &
\end{tikzcd}$$

$\exists$ assignment $= \{\bar{a}\}$

Strategy Extraction from LDQ-Refutations

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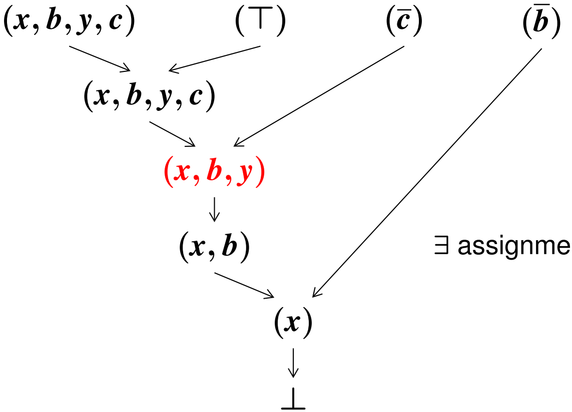

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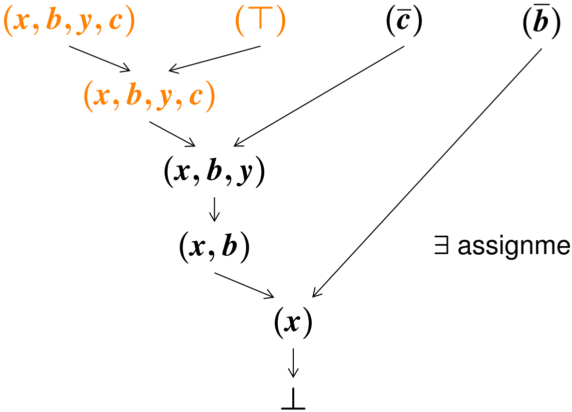

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Strategy Extraction from LDQ-Refutations

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Strategy Extraction

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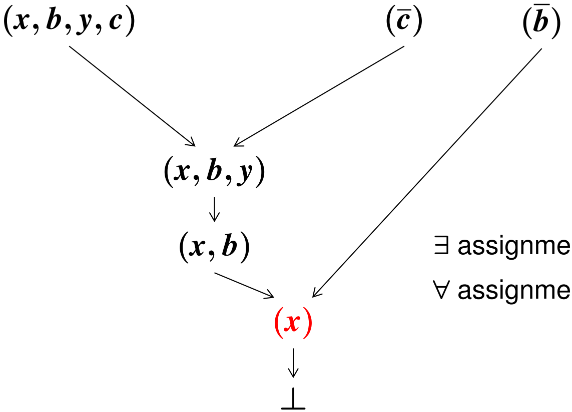
$\exists$ assignment $= \{\bar{a}\}$

$\forall$ assignment $= ?$

Strategy Extraction from LDQ-Refutations

$\forall x\ \exists b\ \forall y\ \exists c$

Strategy Extraction



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$\forall$ assignment $=$?

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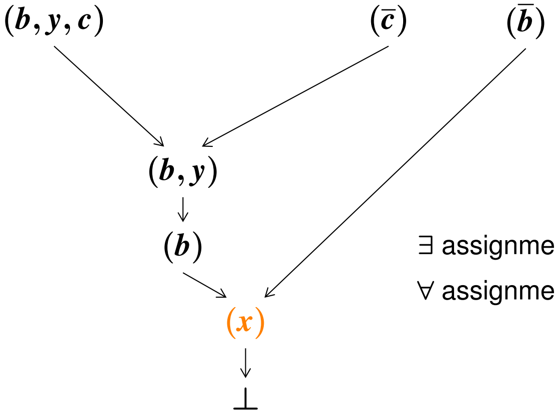

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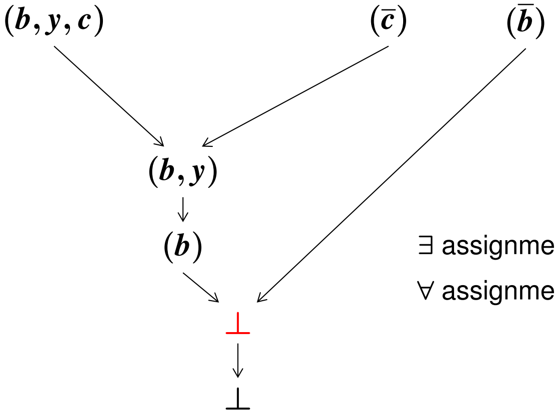

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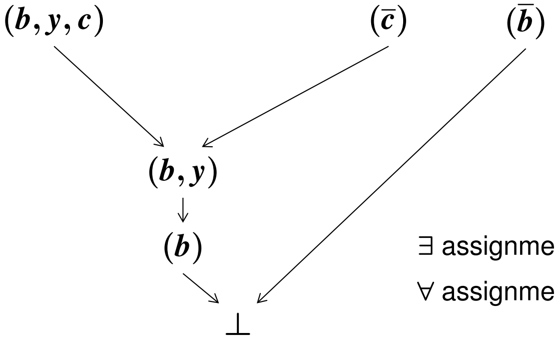

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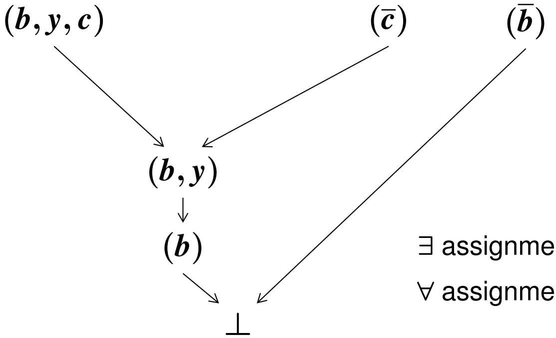

$\exists$ assignment $= \{\bar{a}\}$

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…and so on

Conclusions and Future Work

We have shown

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- Exponential speed-up in refutation size employing LDQ-resolution instead of Q-resolution on $(\varphi_t)_{t\geq 1}$ formulas

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Open issues

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Open issues

- How to extract Herbrand functions from LDQ-refutations
- LDQ-resolution support in QBF solvers

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