

Towards a Logic-Based Framework for Analyzing Stream Reasoning

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FAKULTÄT
FÜR INFORMATIK

Faculty of Informatics





What & Why

“Towards a Logic-Based Framework for Analyzing **Stream Reasoning**”

- ▶ **Stream Reasoning**

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 - ▶ Streams = **tuples** (atoms) with **timestamps**
 - ▶ Essential aspect: **window** functions

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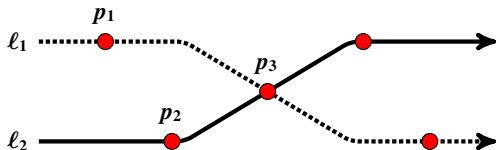
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 - ▶ Streams = **tuples** (atoms) with **timestamps**
 - ▶ Essential aspect: **window** functions
- ▶ Logic-Based: Lack of theory
- ▶ **Analysis**: Hard to predict, hard to compare

Example: Public Transportation Monitoring

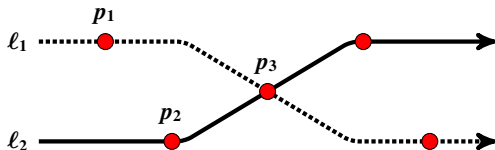


Example: Public Transportation Monitoring

PLAN			
L	X	Y	Z
ℓ_1	p_1	p_3	8
ℓ_2	p_2	p_3	3
...			

LINE	
ID	L
a_1	ℓ_1
a_2	ℓ_2
...	

OLD
a_1
...

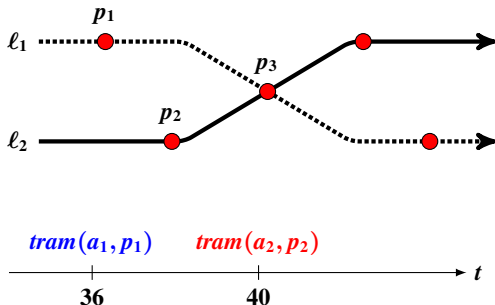


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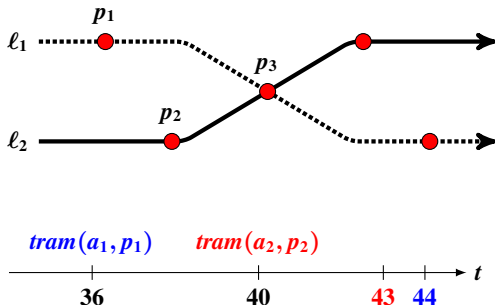


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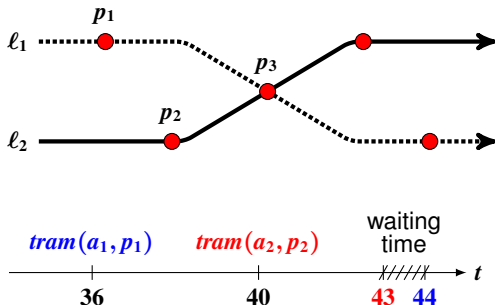
- ▶ Report trams' expected arrival time.

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OLD
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- ▶ Report trams' expected arrival time.
- ▶ Report **good** connections between two lines at a given stop.

Streams

- ▶ Data Stream $D = (T, v)$

$$T = [0, 50]$$

$$v = \{36 \mapsto \{tram(a_1, p_1)\}, 40 \mapsto \{tram(a_2, p_2)\}\}$$

Streams

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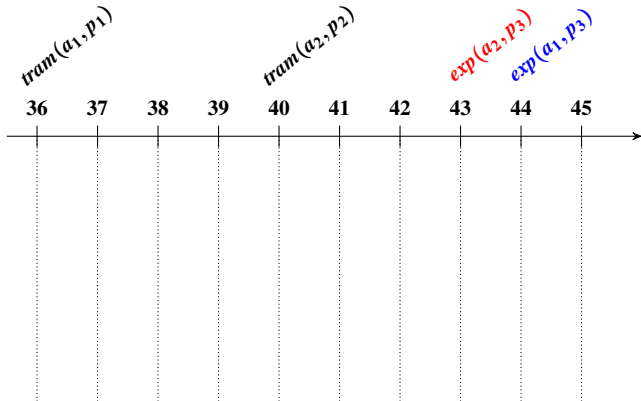
$$v = \{36 \mapsto \{tram(a_1, p_1)\}, 40 \mapsto \{tram(a_2, p_2)\}\}$$

- ▶ Interpretation Stream $S^* = (T^*, v^*) \supseteq D$

$$T^* = [0, 50]$$

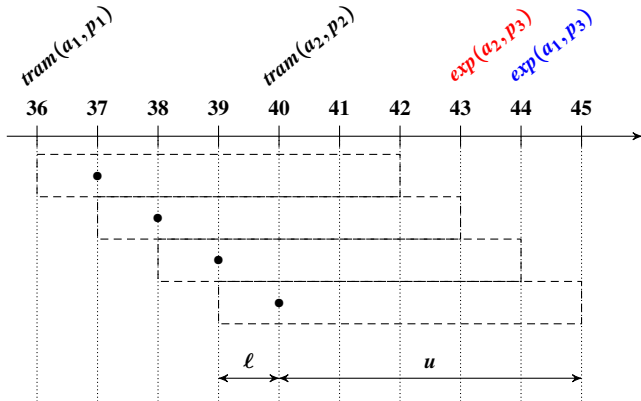
$$v^* = \left\{ \begin{array}{ll} 36 \mapsto \{tram(a_1, p_1)\}, & 40 \mapsto \{tram(a_2, p_2)\}, \\ 43 \mapsto \{exp(a_2, p_3)\}, & 44 \mapsto \{exp(a_1, p_3)\} \end{array} \right\}$$

Window Functions



$$S' = w_L(S, t, \vec{x})$$

Window Functions



$$S' = w_{\tau}(S^*, 40, (1, 5, 1)) = ([39, 45], \left\{ \begin{array}{l} 40 \mapsto \{tram(a_2, p_2)\}, \\ 43 \mapsto \{exp(a_2, p_3)\}, \\ 44 \mapsto \{exp(a_1, p_3)\} \end{array} \right\})$$

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$$\boxed{\vec{x}}_{\iota, ch} \iff w_{\iota}(ch(S^*, S), t, \vec{x})$$

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$$ch_1(S^*, S) = S^*$$

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- ▶ $\boxplus_{\tau}^{10} = \boxplus_{\tau, ch_2}^{10, 0, 1}$ $w_{\tau}(ch_2(S^*, S), t, (10, 0, 1)) = w_{\tau}(S, t, (10, 0, 1))$

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- ▶ nesting of window operators

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- ▶ various ways for time references
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$$\boxplus_{\tau}^{60}\square\boxplus_{\tau}^5\diamond\mathit{tramAt}(p_1)$$

- ▶ but need rules:

$$\mathit{tramAt}(P) \leftarrow \mathit{tram}(X, P).$$

Entailment

- ▶ Structure $M = \langle T^*, v^*, W, B \rangle$

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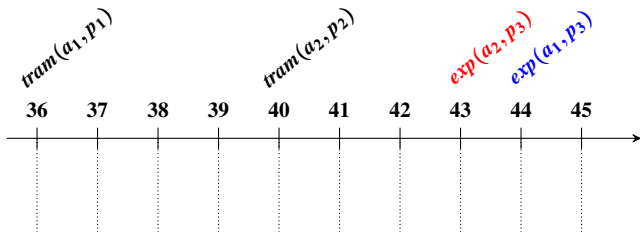
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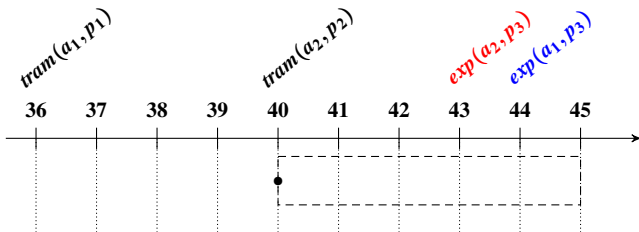
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$M, S, t \Vdash \boxplus_{\vec{t}, ch}^{\vec{x}}\alpha$	iff	$M, S', t \Vdash \alpha$ where $S' = w_{\vec{t}}^{\vec{x}}(ch(S^*, S), t, \vec{x})$.

Entailment: Example



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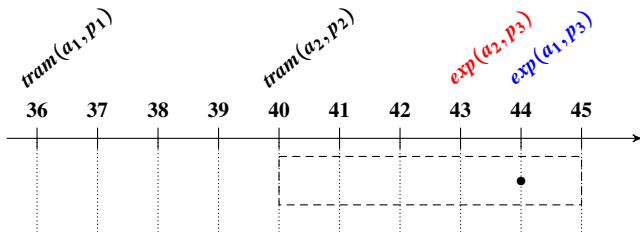


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$$\uparrow$$

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$$\Uparrow$$

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- ▶ Program: set of rules.
- ▶ Example:

$$P = \left\{ \begin{array}{l} @_{Texp}(ID, Y) \leftarrow \boxplus_p^{\text{idx}, n} @_{T_1} \text{tram}(ID, X), \\ \quad \text{line}(ID, L), \text{plan}(L, X, Y, Z), \\ \quad T = T_1 + Z. \\ \\ gc(ID_1, ID_2, X) \leftarrow @_{Texp}(ID_1, X), \\ \quad @_T \boxplus_{\tau}^{+5} \diamond \text{exp}(ID_2, X), \\ \quad \text{not } \text{old}(ID_2). \end{array} \right\}$$

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- ▶ Model: $M, t \models P \Leftrightarrow M, S^*, t \Vdash \beta(r) \rightarrow \alpha$,
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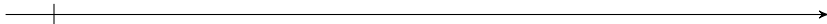
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- ▶ Reduct: $P^{M,t} = \{r \in P \mid M, t \models \beta(r)\}$
- ▶ Answer: M is an answer of P (for D at time t) iff M is a minimal model of $P^{M,t}$

Conclusion Stream

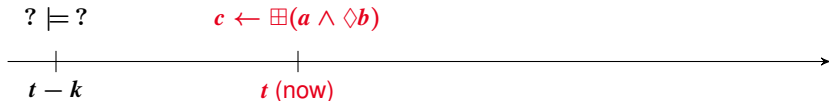
$? \models ?$



$t - k$

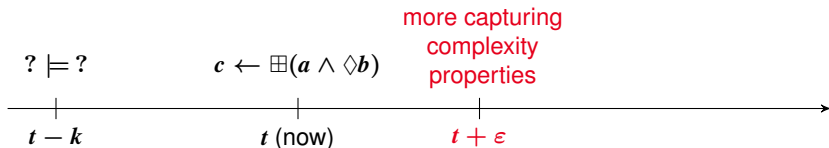
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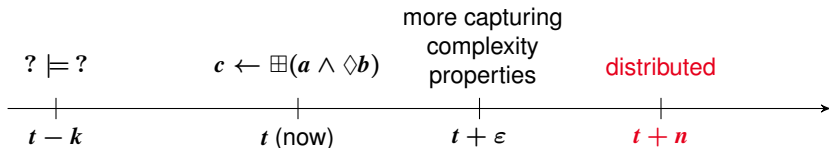
- ▶ Past: Lack of theoretical underpinning for stream reasoning
- ▶ **Now: First language for modelling semantics precisely**
 - ▶ flexible window operator (first class citizen)
 - ▶ time reference / time abstraction
 - ▶ rule-based semantics
 - ▶ more: capturing a fragment of CQL

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- ▶ Eventually: Distributed setting, heterogeneous nodes